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Comparing different hard-thermal-loop approaches to quark number susceptibilities

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Abstract. We compare our previously proposed hard-thermal-loop (HTL) resummed calculation of quark number susceptibilities using a self-consistent two-loop approximation to the quark density with a recent calculation of the same quantity at the one-loop level in a variant of HTL-screened perturbation theory. Besides pointing out conceptual problems with the latter approach, we show that it severely over-includes the leading-order interaction effects, while including none of the plasmon terms, which is the main reason for requiring improved resummation schemes.

1 Introduction

In view of the ongoing search for quark–gluon plasma signals in the early stages of ultrarelativistic heavy-ion collisions, quark number susceptibilities (QNSs) have recently received enhanced attention because of their direct connection with fluctuations of conserved charges which could in principle discriminate against a purely hadronic phase [1–3]. Concurrently, new results for QNS have become available from lattice gauge theory [4,5] which considerably improve upon previous studies [6], and moreover extend them to higher T/T_c. The diagonal[1] QNS are found to increase sharply at the deconfinement phase transition toward a large percentage of the ideal-gas value $\chi_0 = NT^2/3$ for SU(N) and massless quarks.

Conventionally resummed perturbative results for thermodynamic quantities, on the other hand, do not seem to be applicable to account for the observed deviation from ideal-gas behavior because of a complete lack of convergence for all temperatures of interest. In the case of the free energy, this problem is particularly severe, because the so-called plasmon contribution $\propto \alpha_\mathrm{s}^{3/2}$ is larger than the leading-order interaction term $\propto \alpha_\mathrm{s}$ for all temperatures $T \lesssim 10^5 T_\mathrm{c}$. A similar but less dramatic problem also occurs with the perturbative result for the diagonal QNS

$$\frac{\chi}{\chi_0} = 1 - 2\frac{\alpha_\mathrm{s}}{\pi} + 8\sqrt{1+\frac{N_f}{6}}\left(\frac{\alpha_\mathrm{s}}{\pi}\right)^{3/2} + O(\alpha_\mathrm{s}^2 \log(\alpha_\mathrm{s})) \quad (1)$$

for QCD ($N = 3$) with N_f quark flavors. For $N_f = 2$ the plasmon term overcompensates the term $\propto \alpha_\mathrm{s}$ for all temperatures $T \lesssim 40 T_\mathrm{c}$, and only for $T \gtrsim 700 T_\mathrm{c}$ does χ/χ_0 show the expected growth with temperature, starting from values extremely close to the ideal-gas result.

In [7] we have shown that these problems can be avoided by a reorganization of perturbation theory which is based on a self-consistent (Φ-derivable) *two-loop* approximation to the thermodynamic potential [8]. The latter leads to a non-perturbative expression for entropy and quark density which can be used to resum the so-called hard thermal loops (HTL) [9,10] and particular next-to-leading order corrections thereof. The results for QNS thus obtained are monotonic functions of T/T_c which account at least for a sizable part of the deviation from the ideal-gas behavior observed in lattice calculations for $T/T_\mathrm{c} \gtrsim 2T_\mathrm{c}$.

Recently, a different approach to resum the effects of HTL in QNS has been put forward in [11] which starts from quark number charge correlators. Employing HTL propagators and vertices at *one-loop* order, one finds substantially larger deviations from the ideal-gas limit, seemingly in a good agreement with the lattice results of [4].

In view of the large efforts invested at present by lattice gauge theorists to explore the effects of small chemical potentials at high temperature in QCD, we think it worthwhile to explain the fundamental differences between our approach and that of [11] and explain why, in our opinion, the results of the latter are actually misleading. In particular we show that the one-loop results of [11] severely over-include the leading-order interaction effects, while they contain none of the plasmon effects $\propto \alpha_\mathrm{s}^{3/2}$ (which are the

[1] Off-diagonal QNS are strongly suppressed in the high-temperature phase. Still, they give rise to a puzzling discrepancy between recent analytic and lattice calculations. The leading-order effect $\propto \alpha_\mathrm{s}^3 \log(\alpha_\mathrm{s})$ has recently been calculated in [7], implying a numerical value of $\sim 10^{-4}$, whereas the available lattice results are claimed to be consistent with zero within an accuracy of $\lesssim 10^{-6}$ [4]

source of the problems with conventionally resummed perturbation theory). Moreover, we point out a certain technical difficulty that has been overlooked by the authors of [11], but has the effect to render their result ill-defined in a distributional sense.

More importantly even, we comment on a conceptual problem with the approach followed in [11] which arises because the HTL action is no longer used as the effective theory for soft modes, but is used throughout all of phase space. Implicitly the definition of the quark number charge operator is modified such as to no longer conform with the operator employed in lattice calculations.

Before discussing the approach of [11] in detail in Sect. 3, we briefly review the QNS as obtained from HTL-resummed thermodynamic potentials. Section 4 summarizes our conclusions.

2 QNS from resummed thermodynamic potentials

2.1 Generalities

The QNS of a given quark flavor is by definition the response of the quark number density $\mathcal{N}$ to an infinitesimal variation of the associated chemical potential μ,

$$\chi = \frac{\partial \mathcal{N}}{\partial \mu} = \frac{\partial^2 P}{\partial \mu^2} = \beta \int \mathrm{d}^3x \, \langle \rho(0,\mathbf{x})\rho(0,\mathbf{0})\rangle, \qquad (2)$$

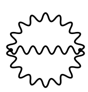
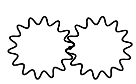
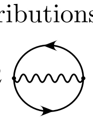
where $P = (\beta V)^{-1} \log Z$ is the thermodynamic pressure, $\beta = T^{-1}$ and $\rho = \bar{\psi}\gamma^0\psi$.

When thermodynamic consistency is automatic, for example in strict perturbation theory to a given order in α_s, it does not matter which of the equivalent expressions on the right-hand side of (2) is employed. However, when further resummations are performed that amount to a partial inclusion of higher-order effects, it does in fact matter. To set the stage we begin by briefly reviewing the approaches which focus on the thermodynamic potential before turning to [11] which starts from the quark number charge correlator.

Expressed as a functional of full propagators (D for gauge bosons and S for fermions, and assuming a ghost-free gauge choice) the thermodynamic potential $\Omega = -PV = -T \log Z$ has the form [12]

$$\beta\Omega[D,S] = \frac{1}{2}\mathrm{Tr}\log D^{-1} - \frac{1}{2}\mathrm{Tr}\Pi D - \mathrm{Tr}\log S^{-1} + \mathrm{Tr}\Sigma S + \Phi[D,S], \qquad (3)$$

where Φ is the sum of two-particle-irreducible "skeleton" diagrams whose lowest-order (two-loop) contributions are

$\Phi[D,S] =$ -1/12  -1/8  +1/2  + ...

As a functional of D and S, Ω is subject to the stationarity condition,

$$\delta\Omega[D,S]/\delta D = 0 = \delta\Omega[D,S]/\delta S, \qquad (4)$$

which is equivalent to

$$\delta\Phi[D,S]/\delta D = \frac{1}{2}\Pi, \quad \delta\Phi[D,S]/\delta S = \Sigma, \qquad (5)$$

for the self-energies Π and Σ. Expressing $\Pi = D^{-1} - D_0^{-1}$ and $\Sigma = S^{-1} - S_0^{-1}$ in terms of bare propagators D_0 and S_0, the representation (3) of course reproduces the ordinary loop expansion.

For example, the leading-order interaction terms $\propto \alpha_\mathrm{s}$ are given by the two-loop diagrams in Φ, whereas single powers of the self-energy insertions in a propagator cancel out in the first four terms of the right-hand side of (3).

Ordinary perturbation theory, however, has infrared problems at finite temperature if the repeated self-energy insertions contained in the term $(1/2)\mathrm{Tr}\log D^{-1}$ are expanded out perturbatively. These can be remedied by a resummation of the leading-order Debye mass

$$\hat{m}_\mathrm{D}^2 = (2N + N_f)\frac{g^2T^2}{6} + \sum_i \frac{g^2\mu_i^2}{2\pi^2} \qquad (6)$$

in the (chromo-)electrostatic propagator, where $g^2 = 4\pi\alpha_\mathrm{s}$ (though new infrared problems arise at order α_s^3). Expanded in powers of g, the resummation of the Debye mass in $(1/2)\mathrm{Tr}\log D^{-1}$ gives rise to the so-called plasmon term in the pressure

$$P_3 = N_g T m_\mathrm{D}^3/(12\pi). \qquad (7)$$

It is this term which is responsible for the dramatic deterioration of the apparent convergence of a perturbative expansion of P in g at finite temperature, and, as remarked in the introduction, to a somewhat lesser degree for QNS which can be derived from the pressure.

2.2 Screened (HTL) perturbation theory

The loss of apparent convergence upon inclusion of the plasmon term in the pressure is in fact generic and also occurs in a simple scalar φ^4 theory [13]. This problem arises as soon as finite-temperature contributions are expanded out in powers of the coupling, which is necessary for the standard ultraviolet renormalization program to become applicable. In order to avoid this, it has been proposed [14] to reorganize perturbation theory by adding screening masses to the classical Lagrangian and to subtract them as counter-terms, but in contrast to the usual resummation program at finite temperature [9,10], this is done for both hard and soft momentum regimes. This in fact alters the ultraviolet structure of the theory, but when combined with a simple minimal subtraction of the additional divergences this resummation appears to significantly improve the apparent convergence of thermal perturbation theory.

In [15,16] this approach has been extended to QCD at one-loop level. It amounts to keeping only the logarithmic terms in (3) and replacing D and S by the HTL propagators,

$$\beta\Omega^{\mathrm{one-loop-HTL}} = \frac{1}{2}\mathrm{Tr}\log\hat{D}^{-1} - \mathrm{Tr}\log\hat{S}^{-1}, \qquad (8)$$

where hatted quantities refer to HTL. If the thermal mass parameters in the HTL are exactly the lowest-order ones, this includes the correct plasmon term (7) without causing the pressure to exceed the ideal-gas value. However, the leading-order interaction pressure $\propto \alpha_\mathrm{s}$ is over-included by a factor[2] of 2.

In order to have both, the leading-order term $\propto \alpha_\mathrm{s}$ and the plasmon term $\alpha_\mathrm{s}^{3/2}$ included correctly, it is necessary to go to two-loop order. Starting from two-loop order, one can turn this so-called "HTL perturbation theory" into a variational perturbation theory, where the HTL action is no longer used as an effective theory for soft modes as in standard HTL resummation [9,10], but just as a gauge invariant mass term which is then eliminated by a principle of minimal sensitivity.

Because of the HTL action involves non-local self-energies and vertices, this optimization of perturbation theory is extremely difficult and has only recently been carried through for QCD to two-loop order [17]. The results are a clear improvement over conventionally resummed perturbation theory as the resummed pressure remains below the ideal-gas limit despite a full inclusion of the contributions through order $\alpha^2 \log(\alpha_\mathrm{s})$. However they appear to account for less than half of the deviation from ideal-gas behavior that is observed on the lattice.

The main virtue of this approach is its completely systematic nature. From a physical point of view, a possible weakness is that the HTL action is used uniformly for soft and hard momenta, whereas the HTL action is accurate only for soft momenta, and for hard ones only in the vicinity of the light-cone. A related problem is that the artificial UV divergences that are introduced involve new subtraction scheme dependences. While these start to be suppressed by powers of α_s only at the (rather forbidding) three-loop order [18], these additional scheme dependences turn out to be numerically rather weak in the two-loop result for QCD.

2.3 HTL resummation of the two-loop Φ-derivable entropy and density

Whereas HTL-screened perturbation is in principle rather generally applicable, we have found, following up an observation made in [19], that specifically for the first derivatives of the thermodynamic potential one can derive remarkably simple expressions from a self-consistent two-loop approximation to the skeleton expansion (3) of the QCD thermodynamic potential. Because of the stationarity property, these derivatives act only on the explicit statistical distribution functions, and not also on those contained in propagators and self-energies. Moreover, after differentiation, the contribution from $\Phi^{\text{two-loop}}$ just cancels part of the second and fourth term on the right-hand side of (3). The derivatives with respect to temperature

[2] As explained in the last paper of [8], [15] had an even stronger over-inclusion due to an inconsistent use of dimensional regularization

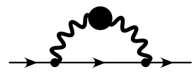


Fig. 1. Next-to-leading order corrections to the asymptotic fermion mass

and chemical potential give entropy and quark densities, respectively, reading [8]

$$\mathcal{S} = -\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial n}{\partial T}\left[\mathrm{Im}\log D^{-1} - \mathrm{Im}\Pi\,\mathrm{Re}D\right] - 2\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f}{\partial T}[\mathrm{Im}\log S^{-1} - \mathrm{Im}\Sigma\,\mathrm{Re}S], \quad (9)$$

$$\mathcal{N} = -2\,\mathrm{tr}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\partial f}{\partial \mu}\left[\mathrm{Im}\log S^{-1} - \mathrm{Im}\Sigma\,\mathrm{Re}S\right], \quad (10)$$

where D and S are determined by one-loop gap equations (5) obtained by restricting Φ to two-loop order.

Although these equations have the form of one-loop expressions involving dressed propagators, they include all the two-loop contributions, but incorporated in the spectral properties of the quasi-particles described by the dressed propagators. This implies that both the leading-order interaction terms $\propto \alpha_\mathrm{s}$ and the plasmon effect $\propto \alpha_\mathrm{s}^{3/2}$ are completely taken into account as soon as D and S are evaluated to sufficient accuracy.

Because the expressions (9) and (10) are manifestly ultraviolet finite, they can be used to resum the effects of HTL without the necessity of subsequent expansions and truncations[3]. At soft momenta, the HTL are valid expressions to the actual full propagators that one would have to use in a self-consistent scheme; at hard momenta it turns out that to leading order the above expressions only probe the vicinity of the light-cone where HTL self-energies remain accurate. The next-to-leading order effect, which is the plasmon effect, turns out to be to one part covered by HTL resummation in the soft regime, and to the remaining part by corrections to the so-called asymptotic thermal masses from HTL-resummed one-loop diagrams.

In the case of the quark number density functional, from which the QNS can be derived, it turns out that all of the plasmon effect is associated with next-to-leading order corrections to the asymptotic fermion mass, shown in Fig. 1.

If only the HTL approximation to the fermion propagator is employed, the quark number density still contains the complete leading-order interaction effects $\propto \alpha_\mathrm{s}$, and – since it need not be expanded out perturbatively – also subsets of higher-order effects, namely those associated with repeated HTL insertions.

In [7] we have evaluated the QNS obtained by taking the derivative of (10) with respect to μ, both in the HTL approximation and in a next-to-leading approximation which incorporates the plasmon effect through the corrections to the asymptotic fermion mass from the diagram of Fig. 1.

[3] For a different approach based on the pressure see also [20]

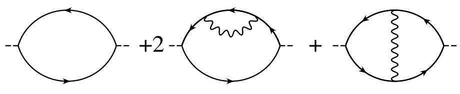


Fig. 2. Lowest-order contributions to (12) in bare perturbation theory

3 QNS from HTL-resummed charge correlators

In [11] an HTL-resummed QNS has been constructed by starting from the charge correlator

$$\chi = \beta \int \mathrm{d}^3x \, \langle \rho(0,\mathbf{x})\rho(0,\mathbf{0})\rangle \equiv \beta \int \mathrm{d}^3x \, \Pi^{>}_{00}(0,\mathbf{x}), \quad (11)$$

where $\Pi^{>}_{\mu\nu}$ is the current–current correlator of a given flavor charge (suitable linear combinations of such quantities give the correlators of electric charge and baryon number).

In Fourier space one has [21–23]

$$\chi = \lim_{k\to 0} \beta \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \Pi^{>}_{00}(\omega,k), \quad (12)$$

where the well-known fluctuation–dissipation theorem allows one to write (in the notation of [10])

$$\Pi^{>}_{\mu\nu}(\omega,k) = -\frac{2}{1-\mathrm{e}^{-\beta\omega}} \mathrm{Im}\Pi^{R}_{\mu\nu}(\omega,k) \quad (13)$$

with $\Pi^{R}_{\mu\nu}$ the retarded response function.

In conventional perturbation theory the first few diagrams contained in (12) are shown in Fig. 2[4]. The first diagram gives the ideal-gas value, and the leading-order interaction term $\propto \alpha_\mathrm{s}$ in (1) is given by the two-loop order diagrams. The plasmon effect $\propto \alpha_\mathrm{s}^{3/2}$ comes from those higher-order diagrams which correspond to repeated self-energy insertions into the gluon lines of the two-loop diagrams of Fig. 2.

Calculating χ from (12) is in fact a bit more involved than starting from the thermodynamic potential as also noticed in [21]. Because charge conservation implies $\omega\Pi^{>}_{00}(\omega,0) = 0$ one has $\lim_{k\to 0} \Pi^{>}_{00}(\omega,k) \sim \delta(\omega)$.

If, for example following HTL-screened perturbation theory, one is interested in the one-loop contribution arising from dressing the propagators in Fig. 2, this will spoil this behavior. In order to have charge conservation one needs to employ HTL vertices in addition to HTL propagators, which is precisely what the authors of [11] have proposed.

However, this raises an important conceptual problem: in effect this use of the HTL vertices replaces the ordinary charge operator $\bar\psi\gamma^0\psi$ by the non-local object $\bar\psi(\gamma^0+\hat\Gamma^0)\psi$ derivable from the non-local HTL action. The correspondingly re-defined QNS is therefore no longer directly related to the quantity defined in (11) and measured in lattice simulations.

[4] One-particle-reducible diagrams, which in principle contribute, vanish because of the tracelessness of color matrices

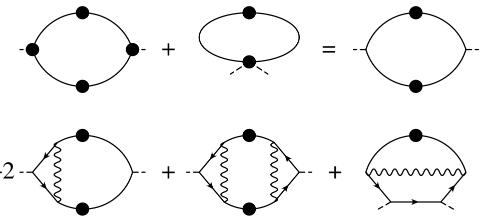


Fig. 3. One-loop contributions to (12) in HTL-screened perturbation theory. The vertex parts built from bare propagators are understood to be evaluated in the HTL approximation

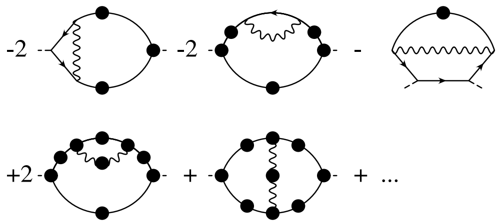


Fig. 4. Two-loop contributions to (12) in HTL-screened perturbation theory which contribute to the leading-order interaction terms $\propto \alpha_\mathrm{s}$

This is in fact a problem only because the HTL action is no longer used as an effective theory appropriate for soft momentum scales, but is used equally for soft and hard momenta. As an effective theory, obtained after integrating out the hard momenta and used for soft modes only, the appropriate charge operator is indeed the non-local quantity involving the HTL vertex $\hat\Gamma^0$. It is this operator which enters in a perturbative matching to the full theory. But replacing the ordinary charge operator by the HTL-dressed one for all momenta clearly corresponds to abandoning the definition (11) for the QNS.

Leaving this issue aside for now, we continue by analysing the diagrammatic content of χ in HTL-screened perturbation theory. Since HTL vertices are strictly one-loop quantities, to one-loop order it is as shown in Fig. 3.

However, while all the topologies that are present in the two-loop diagrams of Fig. 2 also appear in the diagrams of Fig. 3, their combinatorial factors are different.

This shows that in the one-loop HTL approximation to (12) the α_s contributions are all overcounted. The second diagram of Fig. 2 is contained with correct combinatorics in the first diagram of the right-hand side of Fig. 3 but appears another time through the HTL four-vertex; the third diagram of Fig. 2 is seen to be over-included by a factor of 2. Moreover, because the HTL approximation for the undressed self-energies and vertex subdiagrams in Fig. 3 does not provide the complete leading-order terms for hard inflowing momenta, this leads to a further source of incompleteness of the terms $\propto \alpha_\mathrm{s}$.

The authors of [11] do not specify how to extend their approach to two-loop order. It is clear, however, that the correct counting is restored only when the modification of the quark number charge is undone by HTL counterterms and all two-loop diagrams are added. The relevant diagrams for completing the order α_s result are shown in Fig. 4, where the first and the third diagram correspond to

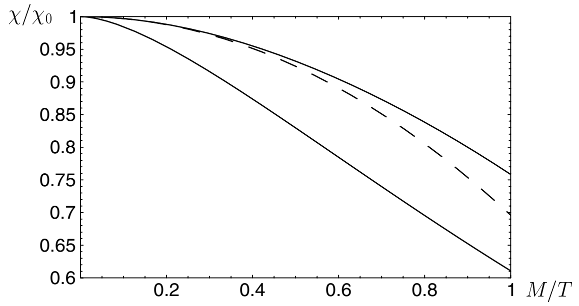


Fig. 5. χ/χ_0 as a function of M/T. The dashed line is the perturbative result to order g^2, the upper full line is the two-loop Φ-derivable approximation evaluated with HTL propagators, the lower full line is the one-loop HTL result of [11]

HTL counter-terms to the charge operator[5]. This means that the definition of the charge operator in (11) has to be modified order by order to approach the standard definition of QNS at least at infinite loop order.

This also shows that the plasmon term $\propto \alpha_s^{3/2}$, which is the reason for seeking improvements of conventionally resummed perturbation theory, only appears in the two-loop order diagrams of HTL-screened perturbation theory, namely through the dressed vector boson lines with a blob. The vector boson lines within the HTL vertices of Fig. 2 are not dressed and thus do not capture anything of the plasmon effect.

We are now in a position to compare with the HTL-resummation approaches discussed in the previous section.

In one-loop HTL-screened perturbation theory along the lines of [15,16] the leading-order interaction term to the thermodynamic potential is over-included by a factor 2, but the plasmon effect is complete (as long as only the leading-order HTL mass parameter is used; including higher-order corrections in the latter would spoil this). The QNS have not been calculated in this approach, but the same pattern would apply, provided the perturbative HTL masses are inserted before[6] differentiating with respect to μ.

On the other hand, in the two-loop Φ-derivable quark density (10), the leading-order interaction term is already correctly included when evaluating it in the HTL approximation but the plasmon term is absent (it arises exclusively from next-to-leading order corrections to the asymptotic (hard) thermal fermion mass).

In Fig. 5, the QNS we have obtained from (10) in the HTL approximation [7] is evaluated as a function of the fermionic plasmon mass[7] M/T and compared with a numerical evaluation of the one-loop HTL result reported in [11]. While our result shows a slightly slower deviation from the ideal-gas limit than the strictly perturbative result to order α_s^1, the result of [11] has considerably stronger deviations because of the over-inclusion of the leading-order interaction term.

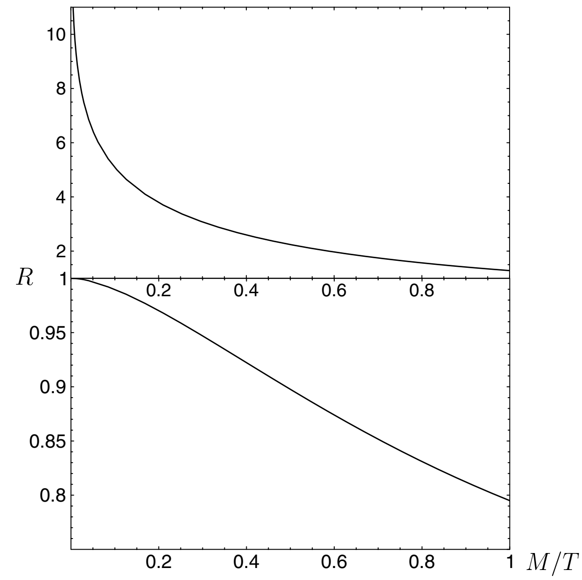


Fig. 6. $R = (\chi - \chi_0)/(\chi^{(2)}_{\text{p.th.}} - \chi_0)$ as a function of M/T for the one-loop HTL result of [11] (upper half of the plot) and for the two-loop Φ-derivable approximation evaluated with HTL propagators (lower half of the plot – note the different scale!). The perturbative result to order g^2 corresponds to the value 1. Only the two-loop HTL result approaches 1 in the limit $M/T \to 0$, where eventually perturbation theory should be reproduced; the one-loop HTL result of [11] is seen to over-include the leading-order interaction effect by a factor which diverges logarithmically as $M/T \propto g \to 0$

In Fig. 6 we consider the expression

$$R \equiv (\chi - \chi_0)/(\chi^{(2)}_{\text{p.th.}} - \chi_0) \tag{14}$$

which measures the deviation of the interaction part of χ from the perturbative result $\chi^{(2)}_{\text{p.th.}}$ to order α_s, and plot the respective results, again as a function of M/T. While our result obtained from (10) in the HTL approximation [7] goes to 1 in the limit of a weakly coupled theory, the effective over-inclusion of the leading-order interaction term of [11] *diverges* in this limit. In fact, it turns out that the result reported in [11] involves a contribution $\propto (M/T)^2 \log(M/T) \sim \alpha_s \log(\alpha_s)$, which does not exist in the correct perturbative expansion. This over-inclusion problem is therefore much more severe than in the case where one-loop HTL-screened perturbation theory is applied to the thermodynamic potential [15,16].

This clearly shows that the one-loop HTL resummation of the charge–charge correlator cannot be compared with either our results, which are based on the two-loop expression (10), or perturbation theory which it seeks to improve upon. In our opinion it is also completely premature to compare with the available lattice results on QNS,

[5] The 2nd and 4th diagram have opposite combinatorial factors but may also contribute to order α_s because of the incompleteness of the HTL approximation for hard loop momenta

[6] Giving up thermodynamic consistency, one could think of identifying the mass parameter in HTL-screened perturbation theory only after this differentiation. This would in fact give a correct leading-order interaction term, but would loose the plasmon effect (and not completely reproduce the terms of order α_s^2 and higher contained in (10))

[7] For plots in terms of α_s or T/T_c, see [7]

because the two-loop contributions of Fig. 4 will have to correct for the enormous over-inclusion of terms $\propto \alpha_s$.

But it appears to be questionable whether a two-loop HTL-screened perturbation theory calculation of (12) is at all practicable. There are in fact certain technical problems with the result reported in [11] already at one-loop order. In (34) of [11] one can see that the result for $\Pi^{>}_{00}(\omega, \mathbf{0})$ is proportional to the integral

$$\int \mathrm{d}^3k \int \mathrm{d}x \int \mathrm{d}x' n_F(x) n_F(x') \rho_+(x,k) \rho_-(x,k) \\ \times \frac{(\omega - x - x')^2 \delta(\omega - x - x')}{\omega^2}, \quad (15)$$

where $\rho_\pm(x,k)$ are the HTL spectral functions for the two fermionic quasi-particle branches of the HTL approximation. In [11] the latter are used to put $x = -x'$, so that the second line of (15) is reduced to $\delta(\omega)$ in conformity with the expectations from charge conservation. However, this term is clearly ill-defined and might with equal justification be put to zero identically as is suggested by the way we have written it. In order to have a well-defined expression, it seems to be necessary to keep the external spatial momentum different from zero and take the limit to zero only after having performed the integral over ω (as demanded by (12)). But that would make its evaluation in HTL perturbation theory a hopelessly difficult task, already at one-loop order.

4 Conclusions

In this paper, we have discussed various possibilities for HTL resummation in the calculation of QNS and have in particular analysed the recent proposal of [11]. We have shown that the resummed one-loop calculation presented there severely over-includes the leading-order interaction terms, while not including anything of the plasmon effect, both of which would be corrected only at two-loop order. Thus only the latter should be viewed as an improvement over ordinary perturbation theory in a comparison with the available lattice results.

However, because of the technical problems mentioned at the end of the previous section, it would seem to be more sensible to calculate the QNS through a two-loop HTL-screened perturbation theory evaluation of the pressure along the lines of [17].

On the other hand, the HTL-resummed calculation of QNS of [7] is based on a two-loop Φ-derivable approximation and does include correctly both the leading-order interaction effect, $\sim \alpha_s$, and the next-to-leading-order one, $\sim \alpha_s^{3/2}$ (together with an infinite series of higher-order effects due to HTL). The results in [7] show the same trend as the lattice results, but a significant difference still remains, which calls for further studies, both on the analytic side, by further improving the resummation schemes, and on the lattice side, by increasing the reliability of the numerical results.

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